LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

THIRD SEMESTER – **APRIL 2023**

UMT 3501 – ABSTRACT ALGEBRA

Date: 02-05-2023 Dept. No. Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

SECTION A

Answer ALL the Questions

1.	**Answer the following:** **(5 x 1 = 5)**		
a)	Define equivalence class of an element in the set.	K1	CO1
b)	Recall order of an element in the group.	K1	CO1
c)	Given two groups G and G', name an example for homomorphism existing between G and G'.	K1	CO1
d)	State the Pigeonhole principle.	K1	CO1
e)	Define Maximal ideal of ring R.	K1	CO1
2.	**Fill in the blanks** **(5 x 1 = 5)**		
a)	Two integers a and b are relatively prime if ______________.	K1	CO1
b)	The center of a group is defined as Z(G) = _________________	K1	CO1
c)	A permutation is said to be even if ________________.	K1	CO1
d)	Two elements a and b are said to be zero-divisors in ring R if _____________.	K1	CO1
e)	Let R be a commutative ring with unit element. Two elements a,b ϵ R are said to be associates if ______________.	K1	CO1
3.	**Choose the correct answer for the following** **(5 x 1 = 5)**		
a)	The greatest common divisor of 1128 and 33 is identified as (i) 3 (ii) 2 (iii) 1 (iv) 5	K2	CO1
b)	If a group G has no non-trivial subgroups, then G is classified as (i) non-abelian. (ii) cyclic. (iii) abelian but not cyclic. (iv) cyclic but not abelian.	K2	CO1
c)	Let $\theta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 3 & 5 & 6 & 4 \end{pmatrix}$. Identify the orbits of 1 and 4. (i) o(1)={1,3} & o(4) = {2,4,5,6} (ii) o(1)={1,2} & o(4) = {3,4,5,6} (iii) o(1)={3,4,5} & o(4) = {1,2} (iv) o(1)={1,2} & o(4) = {4,5,6}	K2	CO1
d)	If U is an ideal of R and 1 ϵ U then identify the right answer (i) U = R (ii) U ⊂ R (iii) R ⊂ U (iv) U = {1}.	K2	CO1
e)	Identify the Encoded message for DOJHEUD using Caesar's Cipher. (i) ALGEBRA (ii) ABELIAN (iii) ABSTRACT (iv) CYCLIC		
4.	**Say TRUE or FALSE** **(5 x 1 = 5)**		
a)	Every element a in group G has an unique inverse in G.	K2	CO1
b)	A subgroup N is normal in group G if and only if $gng^{-1} = n$ for all g ϵ G and n ϵ N.	K2	CO1
c)	Kernel of a homomorphism between groups G and G', is a normal subgroup of G.	K2	CO1
d)	The homomorphism between rings R and R', maps multiplicative identity (unit element) of R to multiplicative identity (unit element) of R'.	K2	CO1

e)	A Euclidean ring need not possess a unit element.	K2	CO1

SECTION B

Answer any TWO of the following **(2 x 10 = 20)**

5.	If a and b are integers, not both 0, then show that gcd(a,b) exists. Moreover, demonstrate that we can find integers m_0 and n_0 such that gcd(a,b) = $m_0a + n_0b$.	K3	CO2
6.	Illustrate "If *G* is a group, then set of all automorphisms on *G* is a group" with proof.	K3	CO2
7.	Produce the proof for statement "a finite integral domain is a field".	K3	CO2
8.	Demonstrate about Public key cryptosystem.	K3	CO2

SECTION C

Answer any TWO of the following **(2 x 10 = 20)**

9	Analyze the following statement and explain the proof. (a) A non-empty subset H of the group G is a subgroup of G if and only if (5 Marks) (i) a,b ϵ H implies that ab ϵ H and (ii) a ϵ H implies that a^{-1}ϵ H.	K4	CO3
	Analyze the following statement and explain the proof. (b) If H is a nonempty finite subset of a group G and H is closed under the operation of G, then H is a subgroup of G. (5 Marks)		
10.	Explain how any group structure is comparable, using Cayley's theorem, with the proof.	K4	CO3
11.	Explain the proof for the following statements. If R is a ring, then for all a,b in R. Then point out that (i) a.0 = 0.a=0 (ii) a.(-b)=(-a).b=-(ab) (iii) (-a)(-b)=ab If in addition, R has a unity 1. Then infer that (iv) (-1)(a) = -a (v) (-1)(-1)=1	K4	CO3
12.	If R is a commutative ring with unity and M is an ideal of R, then infer that M is a maximal ideal of R if and only if R/M is a field.	K4	CO3

SECTION D

Answer any ONE of the following **(1 x 20 = 20)**

13.	Defend Lagrange's Theorem by providing a suitable proof. Given order of the group G = 32 and G'=21. Using Lagrange's theorem, list all the possible order of subgroups for the groups G and G'.	K5	CO4
14(a).	Convince that "any positive integer α > 1 can be factored in a unique way as $p_1^{\alpha_1}.p_2^{\alpha_2} ... p_t^{\alpha_t}$, where $p_1 > p_2 > \cdots > p_t$ are prime numbers and where each $\alpha_i > 0$ by providing a suitable proof. (12 Marks)	K5	CO4
14(b).	Produce a condition on an element in a Euclidean ring to form as an unit and construct a proof to support your condition. (8 Marks)		

SECTION E

Answer any ONE of the following **(1 x 20 = 20)**

15.	Compare the proof of fundamental theorem of homomorphism in groups with the fundamental theorem of homomorphism in rings.	K6	CO5
16(a).	Let R be a commutative ring with unit element whose only ideals are (0) and R itself. Then construct the proof showing that R is a field. (6 Marks)	K6	CO5
16(b).	In an Euclidean ring, every ideal is generated by an element say a_0. Now, produce a condition on a_0 such that the ideal generated by it is an maximal ideal and construct the proof to support your condition. (14		

	Marks)		

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